Exploiting natural dynamics of nonlinear compliance using adaptive oscillators

Exploiting Natural Dynamics of Nonlinear Compliance Using Adaptive Oscillators

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1 Introduction

Compliance became the essential part of locomotion in robotics. Due to ability of storing and releasing energy, compliance can be used for energy efficiency or reducing impact during ground collision and gaining robustness. On the other side, natural/passive dynamics are important because by exploiting such dynamics, energy efficiency will be assured. Therefore it is crucial to understand how compliance changes natural dynamics of a system. After this inspection, natural dynamics exploitation can be more straightforward through developing tools like adaptive oscillators. Such research to exploits natural dynamics of compliant system are reported in [1], [2] and [3].

Intuitively, it is known that using linear compliance will result in efficiency in only one mode. For instance, in mass-spring system for each spring constant, there is only one frequency where system is energy efficient. Using variable compliance is an attempt to overcome this problem and gain efficiency over a range of different set-points or tasks. Great achievements reported using variable compliance in robotics application in [4] and [5]. Another way to overcome this problem is using nonlinearity. Natural dynamics and their multi-modality of efficiency will be discussed in this paper. It seems muscle-tendon units in biological systems are taking advantage of such nonlinearity in their compliance [6]. An adaptive oscillator based the one in [7] is presented in this work. This oscillator is able to exploit natural dynamics of system by shaping desired trajectory through frequency and phase lag.

2 Nonlinear Compliance

The most simple nonlinear compliance can be express by $F = kx^3$, where k is constant and x is position. Let assume that we used this spring in spring mass system. Mass set to 1Kg for sake of simplicity. Equation with initial condition is as follow.

$$\ddot{x} = -kx^3, \quad x(0) = A$$

Solution to this fairly simple system is quite complicated. As we see below, solution belongs to Jacobi elliptic functions. A simple form of this function, JacobiSN(t, I), is illustrated in Figure1.

$$x(t) = A\, JacobiSN\left(\left(\frac{\sqrt{2k}}{2}At + InverseJacobiSN(1, I)\right), I\right)$$

This function is similar to sinusoidal trajectories and can be used in robotics application to generate periodic motions. Interesting point about solution of nonlinear spring-mass system is its natural frequency.

$$\omega_n = \frac{\sqrt{2k}}{2}A$$

It shows that natural frequency of this system is function of its Amplitude. At first, this coupling might consider as disadvantage of this system, but this coupling nicely falling into locomotion domain where animals during accelerating, increase their frequency and amplitude simultaneously.

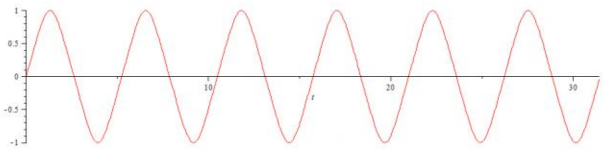

Figure1 JacobiSN(t, I)

Frequency and amplitude coupling for different types of nonlinearity is illustrated in Figure2. These graphs hinting that for every monotonic coupling, there is a nonlinear spring. Finding this nonlinear spring is an open problem.

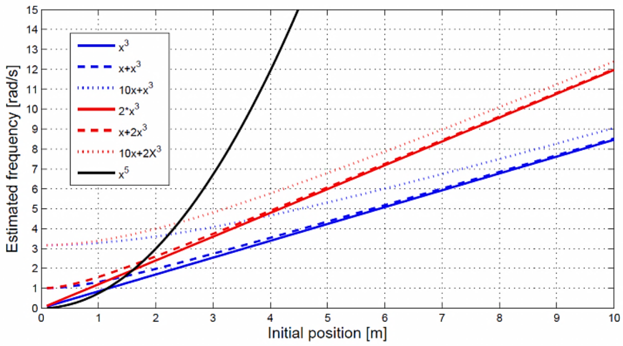

Figure2 Frequency and Amplitude Coupling for different type of nonlinearity

3 Adaptive Oscillator

Adaptive oscillators like Adaptive frequency oscillator [7] are adept tool to converge to natural frequency of external signal. In our work, in order to exploit natural dynamics, this external signal ought to be the controller applied force. Using this approach, block diagram of system is like one shown in Figure3. Note we use trajectory provided by oscillator as desired trajectory for our robotic application. In this system *Plant* can be consider as joint in a robotic manipulator.

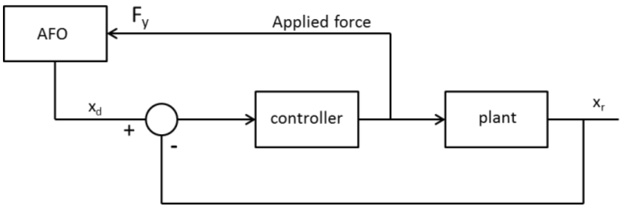

Figure3 Block diagram of AFO

Equations of Adaptive frequency oscillators are as follow:

$$\dot{x} = \gamma\big(\mu - (x^2 + y^2)\big)x - \omega y$$
$$\dot{y} = \gamma\big(\mu - (x^2 + y^2)\big)y + \omega y + \epsilon F_{PID}\,(t)$$
$$\dot{\omega} = -\epsilon F_{PID}(t)\frac{y}{\sqrt{x^2 + y^2}}$$

One of main properties of this oscillator is 90 degree lag between it channels, namely x and y. By assuming periodicity and low tracking error, we can conclude:

$$\begin{cases} \angle\dot{x} \cong \angle y \\ \angle x \cong \angle x_r \end{cases} \rightarrow \ \angle y_{AFO} \cong \angle v_r$$

Like oscillators in [7], this oscillator tries to synchronize y whit external signal, in our case F_{PID}. This synchronization results in:

$$\begin{cases} \angle y_{AFO} \cong \angle F_{PID} \\ \angle y_{AFO} \cong \angle v_r \end{cases} \rightarrow \ \angle F_{PID} \cong \angle v_r$$

This synchronization between applied force and velocity provide the necessary condition for minimum power consumption. In following, we study behavior of this adaptive oscillator for hopper leg shown in Figure4. Oscillation with 0.2m amplitude around rest length of hopper is the desired task for this system (third subplot in Figure5). Note that there is discontinuity in the dynamics between fight and stance phase, resulting in complex natural dynamics for such a simple system.

Proposed adaptive oscillator is used to exploit natural dynamics of this system. Initial frequency and epsilon are 5rads^{-1} and 0.1. Frequency convergence is shown in first subplot of Figure5. This converged frequency in not close to one calculated from $\sqrt{k/m}$. Second subplot in Figure5 shows that AFO apply less force by getting closer to natural dynamics. Another interesting point is that AFO learns to not exert any negative force which can be expected by intuition that jumping up does not imply using downward force. Third subplot in Figure5 shows that tracking performance of controller is satisfying.

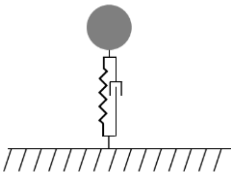

Figure4 Hopper leg; m=0.5kg, k=9Nm^{-1}, b=0.01Nsm^{-1},K_p=100, K_i=0.5,K_d=5.

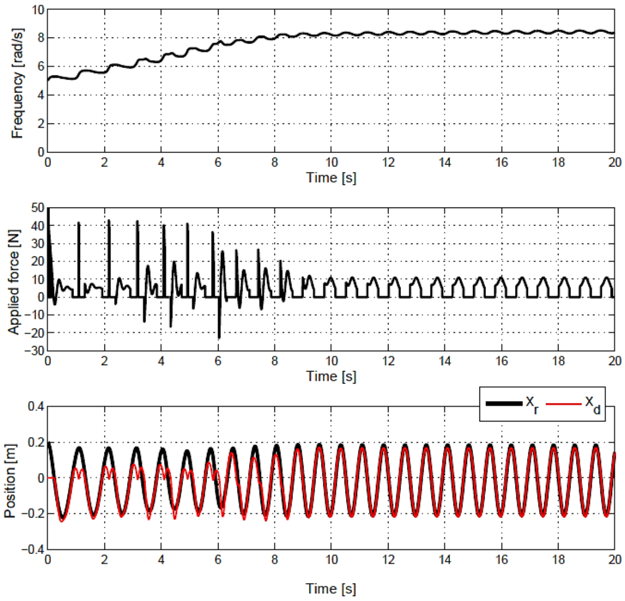

Figure5 Adaptation for hopper leg

4 Natural Dynamics Exploitation

In this section, AFO will be used to exploit natural dynamics of a nonlinear spring for task consisting of ten set points with different amplitude and frequency. A simple choice can be as follow.

$$S_A^{\omega} = \left\{\binom{1}{1}, \binom{2}{2}, \dots, \binom{10}{10}\right\}$$

For measuring performance of particular compliance we define following index which is normalization of average power with square of frequency and amplitude.

$$Index = \sum_{j=1}^{10}\left(\frac{P_j}{A^2\omega^2}\right)^2 \ , \ P_j = \frac{1}{T}\int_0^T \left|f_{pid}\dot{x}\right| dt$$

Performance of different types of nonlinear compliance with no adaptation using sinusoidal trajectories is shown in Figure6. Using nonlinearity can reduced defined index and widen the area around optimum point.

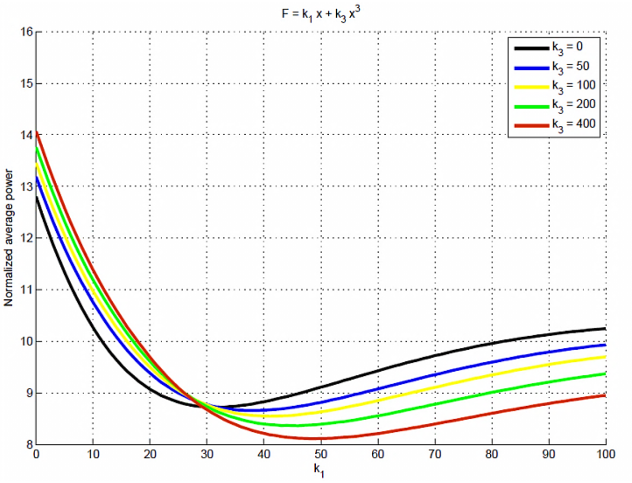

Figure6 Performance of linear and nonlinear compliance for defined task

The other way to solve this multi set-point task is to use nonlinear compliance accompanying with adaptive oscillator to exploit its natural dynamics; most important property of this natural dynamic is frequency-amplitude coupling which is in line with defined task. In following we use a nonlinear compliance with $F = 1.25x^3$ as its equation. We simply set amplitude by multiplying x channel of oscillator by desired amplitude and frequency will be determined by adaptive oscillator. Result for each set-point is illustrated in Figure7. A drift can be observed in in these points, but linear relation is preserved due to good choice of nonlinear compliance. Index for this case is 0.41; comparing this number to lower bound in figure6 shows the efficiency of nonlinear compliance while their natural dynamics are exploited.

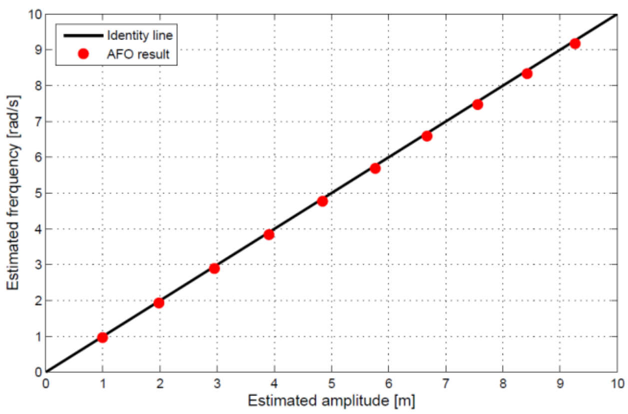

Figure7 Performance of AFO for multi set-point task

5 Open Questions

In this paper, we study natural dynamic of nonlinear compliance in simple systems. Studying complex system like robotic manipulator as general form is in our future work list. A novel and adept adaptive oscillator is presented in this work; capability of this oscillator for natural dynamics exploitation is satisfactory. We built our adaptive oscillators upon Hopf oscillator. For more versatility and learning capability we will use general form as follow where $f(\theta)$ will be learned to exploit natural dynamics.

$$\begin{cases} \dot{\theta} = \omega \\ \dot{r} = f(\theta) \\ \dot{\omega} = g(\theta, r, F_{applied}) \end{cases}$$

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