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Design of a Nonlinear Adaptive Natural Oscillator: Towards Natural Dynamics Exploitation in Cyclic Tasks

Rezvan Nasiri, Mahdi Khoramshahi, and Majid Nili Ahmadabadi

***Abstract*— In this paper, we present the dynamical equations of a nonlinear adaptive natural oscillator (NANO) in order to exploit the natural dynamics in robotic systems. The presented oscillator tries to minimize an energy-based cost function by adapting the shape and frequency of the reference trajectory. Stability, convergence, and optimality of this oscillator are guaranteed analytically. Moreover, the performance of this oscillator is investigated by applying it to three different types of robotic models; i.e., the pendulum, the adaptive-toy, and the hopper-leg.**

***Index Terms*— Energy Efficiency, Cyclic Tasks, Natural Dynamics, Trajectory Learning**

I. Introduction

In robotic locomotion, specially in legged locomotion, energy efficiency, along with velocity and stability, is one of the dominant goals; see [1-3]. In recent years, works on energy efficiency in robotics have gained momentum to address to this issue in most existing legged robots. These works tried to increase the similarity/consistency between the reference trajectory and the natural dynamics of the system by means of manipulating structure (i.e., natural dynamics modification [4-6]) or improving the reference paths (i.e., natural dynamics exploitation [7-9]).

Although reference trajectory optimization is an effective approach for energy consumption reduction, there is still no closed-form and general method. Nevertheless, some works targeted this goal and tried to improve the efficiency by exploring in the feasible trajectory space of the robot; [9-11]. For instance in [10], redundancy is resolved so as to minimize an energy-based cost function.

Passive walkers are very interesting structures which their natural dynamics are modified for cyclic tasks as walking gaits; see [12] as a pioneering work. According to this property, an interesting approach for exploiting natural dynamics in legged locomotion is utilizing the generated trajectories of a passive walker as the reference path for a similar/consistent active robot; see [13] as a remarkable example.

Oscillators such as central pattern generators (CPG) and adaptive frequency oscillators (AFO), are the most favorable toolboxes for exploiting the natural dynamics of robots where a network of oscillators learn a suitable reference trajectory; [14-16]. These pattern generators are applied to a huge number of robots which their initial path is generated(inspired) by(from) their similar/consistent biological structures; [7], [17], [18]. Although the designers hope that these oscillators would exploit the natural dynamics of their robots, there is always some mismatches with biological models and there is no guarantee for optimality of the inspired gaits; see [19]. Dynamic movement primitive (DMP) is another pattern generator which improves an initial pattern by optimizing a desired cost function [9]. This powerful tool is built upon reinforcement learning methods. Similar to conventional learning approaches, this tool optimize the objective function with trail and error; not in an adaptive manner. Recently some works as [20] and [21] tried to design nonlinear adaptive oscillators in order to achieve the energy efficiency. However, lack of convergence and optimality proofs are the main drawbacks of these works.

In [11], we presented an adaptive natural oscillator (ANO) which adapts the frequency of the task in order to reduce the energy consumption. Here, we consider a more general scenario where a *Nonlinear Adaptive Natural Oscillator* (NANO) optimizes the shape of the reference trajectory. To highlight the applicability and generality of NANO in comparison with existing works, we place it under analytical scrutiny in terms of stability, convergence, and optimality.

The rest of the paper is organized as follows. Section II and Section III include problem statement and mathematical analysis where we derive dynamical equations of the oscillator and present proofs for optimality and convergence. Simulations on three different types of robotic models are presented in Section IV, and finally conclusion is placed in the last section.

II. Problem Statement

Consider Fig.1, as the block diagram for a robotic joint. In this block diagram, the joint block represents the dynamical equations of the targeted coordinate. The controller guarantees the bounded tracking error by exerting the applied force/torque (F_a). NANO (described in the following equations) adapts the reference trajectory (x_d) by utilizing the instantaneous applied force as a feedback from the joint.

$$\begin{aligned} &\ddot{x} + K^T\Phi(x) = F_c && , \quad \dot{K} = \epsilon\Phi(x)F_a \\ &F_c = \begin{cases} \frac{\dot{K}^T}{\dot{x}}\int_x^1 \Phi(y)dy & , \quad \dot{x} \neq 0 \\ 0 & , \quad \dot{x} = 0 \end{cases} && , \quad x_d = A_d x \end{aligned} \qquad (1)$$

In NANO's dynamical equations x, $\dot{x}$, and $\ddot{x}$ are the oscillator's coordinate and its first and second time derivatives respectively. ϵ is the adaptation rate which controls the convergence speed. F_c is the "*internal controller*" which guarantees the constant amplitude of oscillations. x_d and A_d

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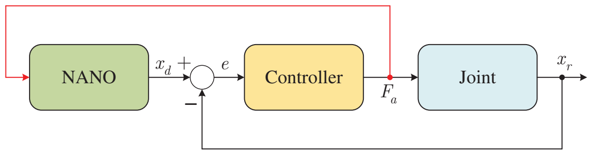

Fig. 1. Control schematic for a robotic joint with adaptive oscillator (NANO) as a pattern generator in control loop.

are the oscillator's output and its amplitude[1]. In addition, $\Phi = [\phi_1\ \phi_2 ... \phi_n]^T$ and $K = [k_1\ k_2 ... k_n]^T$ are the vectors of basis functions and their corresponding coefficients. These coefficients are left to adaptation in order to minimize the squared of instantaneous applied force ($J = F_a^2$) as the cost function. The set of basis functions should be linearly independent w.r.t. x using "*Wronskian Criteria*" (presented in [22, pp. 500]) and satisfy the following inequality.

$$\forall x \neq 0 \rightarrow x\phi_i(x) > 0\ ,\ \phi_i(0) = 0\ ,\ i = 1, 2, .., n \tag{2}$$

In the following section, we drive the dynamical equations presented in Eq.1 and prove the stability, convergence, and optimality of NANO. In addition, we discuss how to select the basis functions[2] (Φ) and adaptation rate (ϵ).

III. Mathematical Analysis

A. Driving oscillator's dynamical equations

To derive the dynamical equations of NANO proposed in Eq.1, we start with a second order dynamical system.

$$\ddot{x} + g(x) = 0\ ;\ \forall x \neq 0 \rightarrow xg(x) > 0\ ,\ g(0) = 0 \tag{3}$$

In this equation, x represents the coordinate of the system, $\ddot{x}$ is the second order time derivative of the coordinate, and $g(.)$ is a nonlinear function. Interestingly this equation can be considered as a mass-spring system with nonlinear spring and unit mass value. Since the presented system has a cyclic behavior, it can be used as a pattern generator.

Proof: [Cyclic motion] To prove the cyclic behavior of Eq.3, consider the following Lyapunov candidate[3].

$$V = \int_0^x g(y)dy + \frac{1}{2}\dot{x}^2 \tag{4}$$

The time derivative of the presented Lyapunov candidate is $\dot{V} = \dot{x}(\ddot{x} + g(x))$ where according to Eq. 3, $\dot{V}$ is identically zero. Hence, based on "*Poincare Bendixson Criterion*" (presented in [23, pp. 61]), Eq.3 has a cyclic behavior. ■

In order to modify Eq. 3 to an adaptive oscillator, we replace $g(x)$ with $K^T\Phi$, where K contains the adaptation parameters[4]. Here, by choosing $\dot{K} = \epsilon\Phi F_a$ as the adaptation rule, the dynamical equations of the oscillator are

$$\ddot{x} + K^T\Phi = 0\ ,\ \dot{K} = \epsilon\Phi F_a \tag{5}$$

[1]For sake of simplicity and without loss of generality, in Section II and Section III, it is assumed that $A_d = 1$. Hence, we have $x \equiv x_d$

[2]For sake of simplicity and without loss of generality in the rest of the paper, we may forbear the argument of parameters for instance $\Phi(x) \rightarrow \Phi$.

[3]We define this Lyapunov function as the "*oscillator's energy*".

[4]$K^T\Phi$ is scalar, hence, we have $K^T\Phi = \Phi^T K$.

B. Force-squared minimization

The presented adaptation rule minimizes the instantaneous applied force.

Proof: [Optimality] Consider the following dynamical equations for a single joint.

$$\ddot{x}_r + f(x_r, \dot{x}_r, w) = F_a\quad ,\quad f : \mathbb{R}^3 \rightarrow \mathbb{R} \tag{6}$$

Where x_r is the position of the joint and w is an external disturbance applied to the joint[5]. $f(.)$ represents all terms of the joint's dynamical equation[6] except $\ddot{x}_r$. The joint's position (x_r) can be written in terms of tracking error (e) and oscillator's coordinate (x) as $x_r = e + x$. Consequently, the joint's acceleration ($\ddot{x}_r$) is $\ddot{x}_r = \ddot{e} + \ddot{x}$. By putting $\ddot{x}_r$ into Eq.6 and utilizing Eq.5, we can rewrite Eq.6 as:

$$F_a = \ddot{e} - K^T\Phi + f(x_r, \dot{x}_r, w) \tag{7}$$

To show that the adaptation rule minimize the instantaneous applied force, we apply the gradient descent rule to the corresponding cost function ($J = F_a^2$) as follows.

$$\dot{K} = -\lambda\nabla_K J = -2\lambda F_a \nabla_K F_a \tag{8}$$

Putting Eq.7 in Eq.8 results in $\dot{K} = \epsilon\Phi F_a$ where $\epsilon = 2\lambda$. ■

The presented proof indicates that the oscillator shapes/adapts the reference trajectory in order to minimize the instantaneous applied force/torque. Interestingly, in dc-motors, instantaneous applied force/torque is proportional with the input current as $I \propto F_a$. Hence, squared of this parameter is proportional with total input power as $P \propto I^2 \propto F_a^2$. Therefore, adaptation in NANO is equivalent with total input power minimization and consequently energy consumption minimization.

C. Internal controller

The presented dynamical equations in Eq.5 cannot guarantee constant amplitude of oscillations during the adaptation. To overcome this problem, we add an extra term (F_c) to Eq.5 as follows.

$$\ddot{x} + K^T\Phi = F_c\ ,\ \dot{K} = \epsilon\Phi F_a \tag{9}$$

The internal controller ensure the constant amplitude; i.e., $(A, \dot{A}) = (1, 0)$. One important note is that the amplitude of oscillations does not appear in the dynamical equations of the oscillator. However, it has a direct impact on the *oscillator's energy* defined in Eq.4. Hence, this energy can be utilized to control the amplitude of oscillations during the adaptation. By replacing $g(x)$ with $K^T\Phi(x)$ in Eq.4, we have:

$$E(x, \dot{x}) = K^T \int_0^x \Phi(y)dy + \frac{1}{2}\dot{x}^2 \tag{10}$$

In *peak-time*[7], the *oscillator's energy* is equal to:

$$E_p = K^T \int_0^A \Phi(y)dy \tag{11}$$

[5]In an n-DOF manipulator, as a general case, it may play the role of other joints' reaction forces.

[6]$f(.)$ is a sufficiently smooth function of x_r, $\dot{x}_r$, and w.

[7]In *peak-time*, we have $x = A$, $\dot{x} = 0$, and $\ddot{x} \neq 0$.

where A is the amplitude of oscillations which changes over adaptation time. The control objective for the *internal controller* is defined as $E(x,\dot{x}) - E_p = 0$. In order to compute F_c so that it satisfies the control objective and guarantees $(A,\dot{A}) = (1,0)$, we take the time derivative of the control objective as follows.

$$\ddot{x}\dot{x} + K^T(\Phi(x)\dot{x} - \Phi(A)\dot{A}) + \dot{K}^T \int_A^x \Phi(y)dy = 0 \quad (12)$$

By substituting $\ddot{x}$ from Eq. 9 to Eq. 12 and by considering $(A,\dot{A}) = (1,0)$, F_c is calculated as follows.

$$F_c = \frac{\dot{K}^T}{\dot{x}} \int_x^1 \Phi(y)dy \quad , \quad \dot{x} \neq 0 \quad (13)$$

Clearly, F_c is not defined for $\dot{x} = 0$. However, the continuity of F_c around the *peak-time* (i.e., $\dot{x} = 0$) can be guaranteed using "*Hopital Rule*" as follows.

$$\lim_{t\to t_p} \frac{\int_x^A \Phi(y)dy}{\dot{x}} = \lim_{t\to t_p} \frac{\Phi(A)\dot{A} - \Phi(x)\dot{x}}{\ddot{x} \neq 0} = 0 \quad (14)$$

D. Convergence proof

Considering the dynamics of the oscillator (Eq.1), dynamics of the system (Eq.6), and the *perfect-tracking*[8] assumption ($e \equiv 0 \to \ddot{e} \equiv 0$), the applied force can be represented as:

$$F_a = f(x,\dot{x},w) - K^T\Phi + F_c \quad (15)$$

By replacing F_c from Eq.1 in Eq.15, we have:

$$F_a = \frac{f(x,\dot{x},w) - K^T\Phi}{1 + \epsilon\Phi^T\Omega} \;\; ; \;\; \Omega = \frac{\int_1^x \Phi(y)dy}{\dot{x}} \quad (16)$$

For small adaptation rates (ϵ), $\epsilon\Phi^T\Omega$ is negligible compared to 1. This simplification leads to:

$$F_a = f(x,\dot{x},w) - K^T\Phi \quad (17)$$

By substituting the calculated applied force in the adaptation rule ($\dot{K} = \epsilon\Phi F_a$), we have the adaptation dynamics as follows.

$$\dot{K} = -\epsilon\left(\Phi\Phi^T K - \Phi f(x,\dot{x},w)\right) \quad (18)$$

Under the assumption of *perfect-tracking*, we can ensure that the joint's trajectory is periodic. This enables us to apply the "*Averaging Theory*" ([23, pp. 402]) to the dynamics of the adaptation as follows[9].

$$\dot{K}_{avg} = -\epsilon\Lambda\left(K_{avg} - \tilde{K}\right) \quad (19)$$

$$\tilde{K} = \Lambda^{-1} \int_{-1}^1 f(y,\dot{x},w)\Phi(y)dx \quad (20)$$

$$\Lambda = \int_{-1}^1 \Phi(y)\Phi^T(y)dx$$

where Λ is a positive definite matrix which is invertible if the bases are linearly independent. Clearly, the adaptation of K depends of the dynamical properties of the system; i.e., $f(x,\dot{x},w)$. In a particular class of single-coordinated systems where $f(x,\dot{x},w)$ can be decomposed as $f(x,\dot{x},w) = k(x) + b(\dot{x}) + h(w)$, we have:

$$\tilde{K} = \int_{-1}^1 k(y)\Phi(y)dx + (b(\dot{x}) + h(w)) \int_{-1}^1 \Phi(y)dx \quad (21)$$

where, based on Eq.2, the second term is identically zero. Hence, K is a constant vector.

To study the stability of the averaged dynamical equation (Eq.19), the "*Lyapunov Theorem*" is used with the following quadratic Lyapunov candidate.

$$V = \frac{1}{2}\Delta K^T \Delta K \;\; ; \;\; \Delta K = K_{avg} - \tilde{K} \quad (22)$$

Taking the time derivate of this Lyapunov candidate and using the averaged dynamical equation (Eq.19) result in:

$$\dot{V} = \Delta K^T \Delta\dot{K} = -\epsilon\Delta K^T \Lambda \Delta K \quad (23)$$

where $\dot{V}$ is a negative definite matrix. Therefore, based on "*Lyapunov Stability Theorem*", the averaged system is globally exponentially stable, and consequently the dynamical equations of the oscillator are on average globally exponentially stable. An interesting conclusion in this convergence proof is Eq.20 which can be considered as an *offline* method to calculate the basis coefficients. However, this *offline* approach is appropriate only if the dynamical properties of the system are fixed and well-known.

E. Selection of the basis functions & the adaptation rate

Based on Eq.20, $\tilde{K}$ is the projection of the joint's dynamical equations (natural dynamics) onto basis functions. It can be shown that $\tilde{K}$ minimizes the cost function ($J = F_a^2$) over a cycle[10]. Therefore, based on Section III-D, convergence of K to $\tilde{K}$ indicates that this oscillator generates cyclic motions which are highly consistent with the natural dynamics of the joint; i.e., natural dynamics exploitation. However, the proper selection of the basis functions is necessary to ensure that such exploitation is feasible and effective.

Previous knowledge about the dynamics of the system can be employed for the basis function selection. For instance, in a mass-spring system with linear compliance, the sinusoidal motion is the most compatible trajectory with natural dynamics of the system. Therefore, in order to have a sinusoidal trajectory, a simple linear basis function ($\Phi = [x]$) is chosen. In this case, the adaptation of the linear basis coefficient is equivalent to the frequency adaptation. As another example, to have a parabolic cyclic motion we can select a sign-function ($\Phi = [sign(x)]$). Interestingly, parabolic motion is the nature of the ballistic motions. In the cases where there is no pre-knowledge about dynamical equations of the system, a set of general approximators such as polynomial bases ($\Phi = [x\ x^3 ...\ x^n]^T$), is an appropriate selection.

According to Eq. 18 and Eq. 19, increasing ϵ results in higher convergence speed. However, this causes ripple

[8]Under this assumption we have $x \equiv x_r$, $\dot{x} \equiv \dot{x}_r$, and $\ddot{x} \equiv \ddot{x}_r$.

[9]Note that a period of time is equivalent to a cycle of oscillation between -1 and 1 for x.

[10]Using the reformulation of F_a from Eq.17, it can be easily checked that the partial derivate of the integrate of the cost function over a cycle of position at $K = \tilde{K}$ is identically zero.

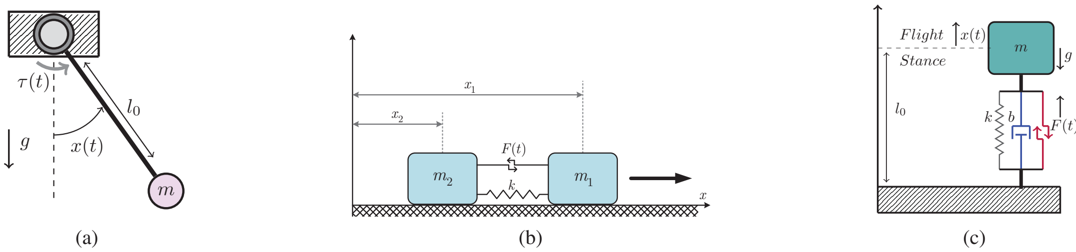

Fig. 2. The simulation setups for NANO. In all setups, gravity acceleration is considered as $g = 9.81m/s^2$ and the controller is a simple PID which is fine tunned for each case. In each setup, initial values of the system's position and velocity are set at the oscillator's output. (a) The pendulum system with a rotary actuation system, a massless rod, and a point mass bob. In this setup, the mass of the bob is $m = 1Kg$ and the link length is $l_0 = 1m$. (b) The adaptive-toy system with a linear actuation system, unequal forward and backward frictions, and a linear spring. Forward and backward friction forces are $F_{fk} = 0.1N$ and $F_{bk} = 0.2N$ respectively, the spring stiffness and its rest length are $k = 8N/m$ and $l_0 = 1m$, and the first and second mass values are $m_1 = m_2 = 1Kg$. (c) The hopper-leg system with a linear actuation system, a linear spring and damper, and a massless leg. The spring stiffness and its rest length are $l_0 = 1m$ and $k = 50N/m$. In this model, the damper coefficient is equal to $b = 0.1Ns/m$ and the mass is $m = 1Kg$.

amplification about the equilibrium point of the averaged system ($\tilde{K}$ in Eq. 19). Therefore, a trade-off between the amplitude of oscillations and the convergence speed should be borne in mind for tuning this parameter. In addition, for further improvement the time varying adaptation rates can be considered as an alternative.

IV. Simulation

In order to investigate the behavior of NANO in terms of adaptation, we apply the presented adaptive oscillator to three different types of dynamical systems: "*pendulum*", "*adaptive-toy*", and "*hopper-leg*". **(1)** First to get an insight, we start with the *pendulum* model and we try to exploit the natural dynamics of this nonlinear single-coordinated system. In this case, we study the effect of chosen basis selection on energy consumption and convergence behavior. **(2)** In the second case, in order to investigate the applicability of the presented pattern generator in simple locomotor systems, we choose *adaptive-toy* [24]. In this simple crawling system, friction is considered as an external disturbance. **(3)** Finally, the presented adaptive oscillator is applied to a *hopper-leg* (see [25]) where discontinuity of contact forces complicates the natural dynamics. Parameters of each system and controllers are described in Fig.2 and its caption in detail.

For each simulation a PID-controller is used in order to control the system on the reference trajectory generated by NANO. The controller gains (k_p, k_i, and k_d) and the adaptation rate (ϵ) are fine-tuned and reported (in Fig.3) for each simulation. Moreover, the initial states of the systems are set on the reference trajectory. Finally, the desired amplitude for *pendulum*, *adaptive-toy*, *hopper-leg* is set to $A_d = 1rad$, $A_d = 1m$, and $A_d = 20cm$ respectively. The simulations are performed using MATLAB/SimMech toolbox [26].

A. The pendulum system

In this simulation, we study the effects of basis functions on the NANO's performance in terms of energy consumption, tracking error, and convergence behavior. To do so, we consider two different types of basis functions. **(1)** A set of polynomial basis functions ($\Phi = [x\ x^3]^T$) as a blind, but appropriate approximators when the dynamics of the system are unknown. (2) Sinusoidal function ($\Phi = [\sin(x)]$) as the consistent structure assuming the dynamics of the pendulum are known[11].

The simulation results for these two different set of basis functions are presented in Fig3a-d. As can be seen in Fig.3a, for both cases, the controller applied force is minimized in course of adaptation. However, using sinusoidal basis function results in zero applied force due to the exact consistency with natural dynamics of this model. Fig.3b shows the convergence of the basis functions to their optimal values. Interestingly, the adapted coefficient of the sinusoidal basis function is exactly equal to g while the coefficients of the polynomial basis functions are converged to their correlated coefficients in "*Taylor Series*" of $g\sin(x) = gx - gx^3/6$.

The consumed energy in course of adaptation is plotted in Fig. 3c. According to this figure, using sinusoidal basis function results in lower energy consumption in comparison with the other case. Finally based on Fig.3d, for both of these cases, tracking performance improves during adaptation, however, sinusoidal basis function leads to zero tracking error (i.e., perfect tracking) due to its consistency with natural dynamics of the system.

B. The adaptive-toy system

In this simulation, we investigate if our proposed adaptive oscillator can provide adaptive and energy-efficient gait for locomotory systems. For this purpose, we study the *adaptive-toy* system with a linear spring as depicted in Fig. 2b. In order to have a forward locomotion, forward friction force is chosen to be less than backward friction. Note that friction forces are external disturbances for this model and they have a nonlinear nature.

Adaptation results of NANO using a linear basis function ($\Phi = [x]$) are illustrated in Fig3e-h. According to Fig. 3e, during the adaptation, the controller applied force is minimized to a residual value which is caused by friction. Fig.3f shows that the adaptive coefficient, on average, converges to $2k/m$. The position of the first and second masses are presented in Fig. 3g. The tracking error is minimized due

[11]The dynamics of the pendulum are explained by $\ddot{x}_r + g\sin(x_r) = F_a$.

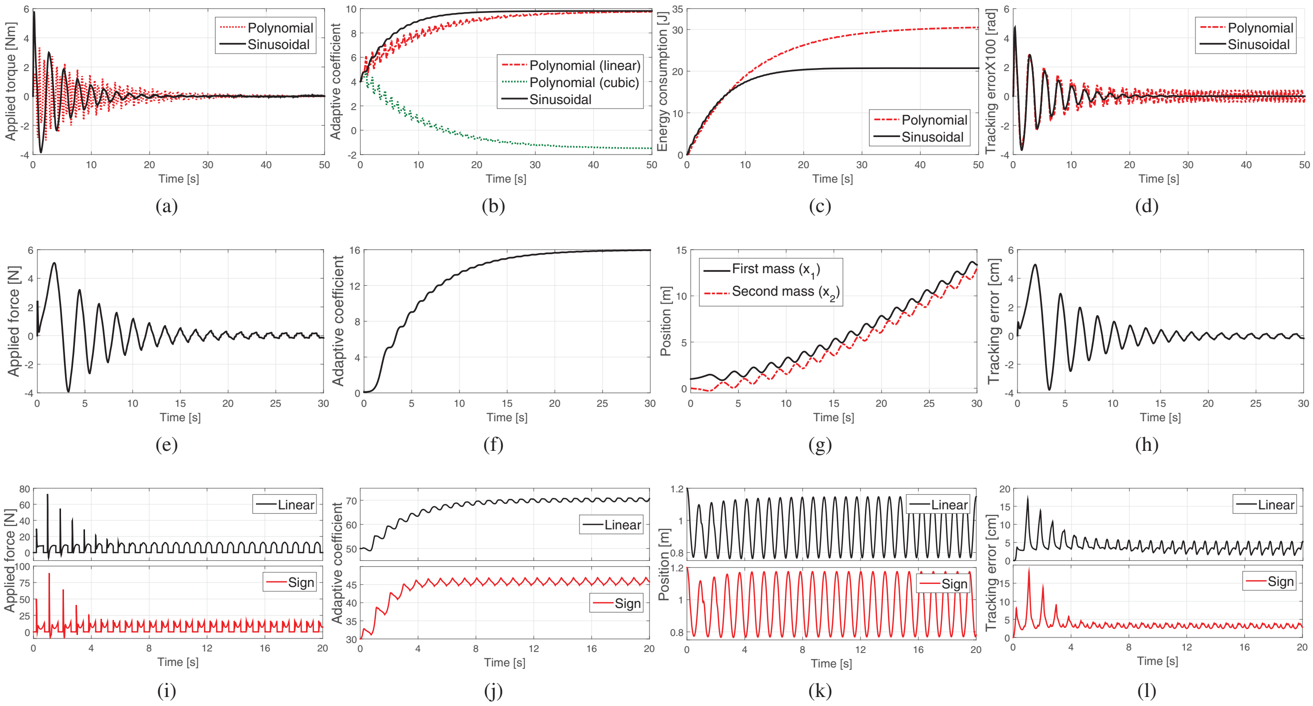

Fig. 3. (a-d) Convergence behavior of the NANO in Pendulum system. For the set of polynomial basis functions, the adaptation rate is set to $\epsilon = 3$ and for sinusoidal basis function it is $\epsilon = 0.5$. Also PID controller gains, are: $k_p = 100$, $k_i = 1$, and $k_d = 10$. Initial values of the adaptive coefficients for all of the cases are 4. (e-h) Convergence behavior of the NANO in adaptive-toy system. For this simulation, the adaptation rate is set to $\epsilon = 1$ and PID controller gains are: $k_p = 100$, $k_i = 1$, and $k_d = 10$. Initial value of the adaptive coefficient is set to 0.1. (i-l) Convergence behavior of the NANO in hopper-leg system. In this simulation, for both of the basis functions, the adaptation rate is set to $\epsilon = 0.5$ and PID controller gains are set to: $k_p = 300$, $k_i = 1$, and $k_d = 40$. Initial value of the adaptive coefficient for the sign basis function is set to 30 and for the linear one is set to 50.

to the natural dynamics exploitation as shown in Fig.3h. In other words, adaptation of NANO make the trajectory more consistent with the natural dynamics of the system which facilitate the controller's task and consequently minimize the tracking error in course of adaptation. The presented results in this simulation indicate that utilizing NANO result in an energy efficient and sufficiently fast motion; i.e., lower cost of transportation (COT).

C. *The hopper-leg system*

To study the efficacy of our adaptation mechanism in legged locomotion system, we consider the *hopper-leg*. Despite its simplicity, this system has troublesome feature; e.g., the contact forces and the hybrid dynamics. The presented hopper-leg has a hybrid dynamical equation between flight and stance phases. The system has a mass-spring motion in the stance phase and a ballistic motion in the flight phase. Also in the flight phase, we have no control on the system's motion. Similar to Section IV-A, here, we compare the performance of NANO for two types of basis functions: linear ($\Phi = [x]$) and sign ($\Phi = [sign(x)]$) basis functions where the former is consistent with the mass-spring motion whereas the latter is consistent with the ballistic motion.

Fig3i-l presents the simulation results on the *hopper-leg* system. Fig. 3i shows that adaptation in both cases results in a lower control effort; i.e., natural dynamics exploitation. In addition, unreported results show that the energy consumption for both of the basis functions are approximately equal. Fig. 3j shows that the coefficients of both basis functions converge to their optimal values. As can be seen in Fig.3k, both of the basis functions generate satisfactory cyclic hopping patterns. Note that, according to Fig. 2c, heights greater(less) than $1m$ are considered as flight(stance) phase. Fig. 3l shows that the adaptation also reduces the tracking error. The results concludes that the linear basis function leads to a smoother applied force whereas the sign function leads to a better tracking error.

V. Conclusion

In this paper, we presented a nonlinear adaptive natural oscillator (NANO) where its dynamical equations were extracted based on a simple mass-spring system. The output trajectory of this adaptive oscillator is a function of arbitrary nonlinear basis functions. We analytically proved that the coefficients of this adaptive oscillator, on average, adapt to their optimal values. Also, it was shown that adaptation of the NANO's coefficients are equivalent with instantaneous applied force minimization. In other words, the adaptive oscillator reduces the energy consumption rendering it natural dynamics exploiter.

To check the performance of the proposed adaptive oscillator, we considered three different types of dynamical models: the *pendulum*, the *adaptive-toy*, and the *hopper-leg*. In the first simulation, we showed how proper selection of basis function plays an important role in the efficacy of the adaptation and energy consumption reduction. In the second system (i.e., the *adaptive-toy*), we showed how the proposed tool can be used to achieve energy efficiency in locomotory systems. In the final example, we showed that our method is applicable to complex systems where the adaptation exhibited robustness to ground contact forces and hybrid dynamics of the system.

The effect of basis functions on the adaptation mechanism is of particular interest in NANO. Different selection of the basis functions enable the NANO to exploit "*differently*" the natural dynamics of the system. For instance, based on Section IV-A, the proper selection of the basis function (consistent with natural dynamics of the system) can lead to a drastic improvement in the controller applied force, tracking error, energy consumption, and convergence behavior. However, such improvements require prior knowledge about the dynamics of the system which is generally unavailable. Hence, in most cases, a set of polynomial basis functions is a proper selection since it is a general function approximator. Moreover, the results of Section IV-C showed a linear profile leads to a smoother applied force whereas a nonlinear profile leads to a lower tracking error. Therefore, selection of the basis function can also be considered as a trade-off between satisfactory tracking performance and practical applied force.

In our future work, we intend to extend our proposed method to general n-DOF systems such as quadrupedal robots. By adapting the joints' trajectories to the natural dynamics of the the robot, we are planning to achieve energy-efficient gaits (e.g., walking and trotting).

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